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Modelization of an experimental solar test box equipped with a water-flow based window

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Abstract

A study on a water-flow window installed in a test box is presented. This window is composed of two glass panes separated by a chamber through water flows. The flow of water comes from an isolated tank which contains heat water. In order to fully evaluate the water-flow window performance for different room and window sizes, locations and weather conditions, a mathematical model of the whole box is needed. The proposed model, in which conduction heat transfer mechanism is the only considered, is one dimensional and unsteady based upon test box energy balance. The effect of the heat water tank, which feeds the water-flow window, is included in the model by means of a time delay in the source term. Although some previous work about moving fluid chamber has been developed, air was used as heat transfer fluid and no fluid storage was considered. Finally a comparison between the numerical solution and the obtained experimental data is done.

1 Introduction

Water-flow based windows are a kind of dynamic windows in which water flows across the chamber formed by two glass panes. This kind of windows can contribute to improve the energy efficiency of buildings [1] in line with European environmental regulations [2].

In order to evaluate the performance of this kind of windows a small test box has been built and it is being tested under real weather conditions at E.T.S. Architecture at Technical University of Madrid.

Figure 1: Test box and water chamber filling process. Source: Own archive.

Since there is no commercial building simulation software available able to reproduce the effect of such *smart windows*, a mathematical model has been developed to evaluate it.

2 Physical problem

In water-flow windows, there are several heat transfer mechanisms involved, as described in figure 2. The water-flow window has a double effect over the air inside the test box. On the one hand, the water absorbs part of the solar radiation that flows across the water-flow window into the box, and on the other hand it provides or removes heat, depending on the circulating water temperature, from the inside air.

A typical working cycle for the experimental assembly in winter season would be the following:

1. Before the sunrise the air inside the box is cold, but since the non transparent walls are properly isolated with extruded polystyrene (XPS), once the sun goes up it makes that inside temperature increases.

2. When the temperature inside the box is higher than a preset comfort temperature (e.g. 21 ℃), *cold water* stored in an isolated tank flows into the water-flow window.

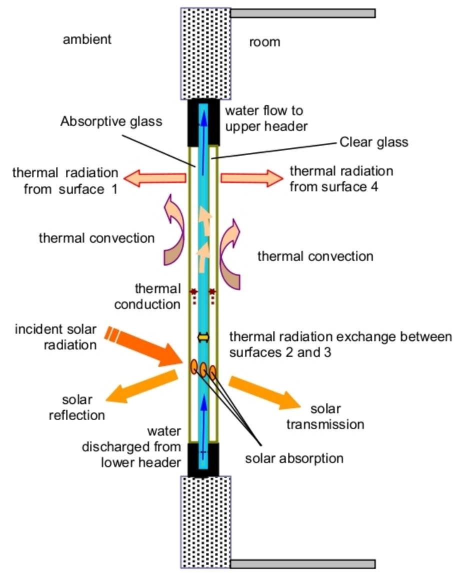


Figure 2: Heat transfer mechanisms involved in a water-flow window. Source: Chow [1].

3. Due to sun radiation and the increase of the air temperature inside the box, the water rises its temperature along sunlight hours. In such way, the water removes heat from the air inside the box at the same time it absorbs a part of the incoming solar radiation over the window.

4. Some time after the sunset, the temperature of the air inside the box decreases and then *hot water* stored in the isolated water tank flows across the window chamber.

5. At a certain night time, circulating *hot water* is not enough to maintain comfort temperature, then an electric heater located inside the box is turned on for the necessary time to maintain the comfort temperature. Due to the water flow in the window chamber, which is hotter that outside air, the use of the electric heater during night time is reduced when compared with a common window.

In order to evaluate the performance of the water-flow window, several temperature sensors are installed in different points of the system.

3 Mathematical approach

Considering the complexity of taking into account all the heat transfer mechanisms seen in figure 2 at the same time, some simplifications have been carried out in the development of the mathematical model for the sake of simplicity:

- only conduction heat transfer mechanism along the solid-fluid layers that shape the box has been developed as starting point,
- physical properties of materials remain constant for the temperature intervals considered,
- only nocturnal data have been compared to avoid solar radiation effect.

3.1 Heat transfer equations

The heat transfer equation for an incompressible fluid, neglecting the pressure effects and viscous stresses is the following

$$\rho\, c_p \left(\frac{\partial T}{dt} + \vec{u}(\nabla \cdot T) \right) = \bigtriangledown \cdot (k \cdot \nabla T) + \dot{q} \tag{1}$$

since water flowing velocity is very slow, convection term can be neglected from equation (1), so it can be reduced to

$$\rho\, c_p \frac{\partial T}{dt} = \bigtriangledown \cdot (k \cdot \nabla T) + \dot{q}. \tag{2}$$

3.2 One-dimensional model

We will consider a one-dimensional model associated to our experimental box (see figure 3). Taking equation (2) as a starting point, an one-dimensional model has been developed. Since heat transfer conduction is considered in all the *physical layers* of the test box, some changes need to be done. Firstly, all physical properties as ρ, cp and k depend on the material, therefore their values change for each domain (Ω). Besides air temperature not only changes along the space but also along the time. In order to model the influence of the heat water flow across the window chamber, a time delay (s) has been added in the temperature source term. This *time delay* corresponds to the total time that a particle needs to go all over the hydraulic circuit. Thus the simplified one-dimensional problem can be described in the following way

$$\rho(x) c_p(x) \frac{\partial T}{\partial t}(x,t) - \frac{\partial}{\partial x} \left(k(x) \frac{\partial T}{\partial x}(x,t) \right) = f(x,t,s,T(x,t-s)) \quad (x,t) \in \Omega \times (0,\infty), \tag{3}$$

where $\Omega =]0, L[$ with L the length of the box cross section. Also, we consider the following initial and boundaries conditions

$$I.C. \qquad T(x,t) = T_0(x,t), \quad x \in \Omega, \quad t \in (-s, 0)$$

$$B.C. \qquad T(0,t) = T_{ext}(t), \quad T(L,t) = T_{ext}(t), \quad t > 0$$

where T is the air temperature inside the box and T_{ext} is the outside temperature.

It should be noted that, in order to avoid modeling the thermally isolated water tank, the equation *source term* $f(t, s, x, T(x, t-s))$ must be referred to the period of time that water needs to travel from the tank to the water-flow window, which is called *time delay* (s). The *source term* must include two terms, the energy provided by the electric heater (when activated) and the energy contribution of the flow of water (that includes the time delay term) by the window chamber at a certain temperature.

On the other hand, the *layers* of the test box are described in figure 3.

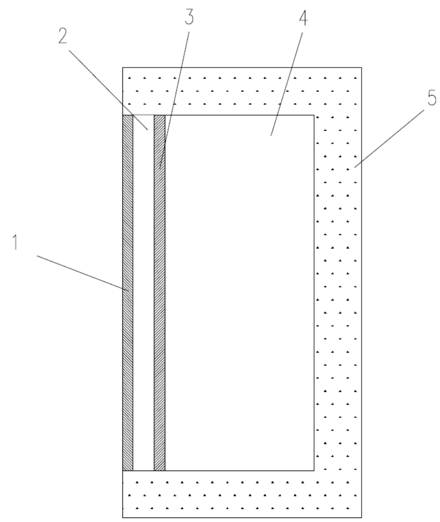


Figure 3: Non scaled bi-dimensional scheme of the test box.
1. Outer glass pane, 2. Circulating water,
3. Inner glass pane, 4. Inside air,
5. Insulation.

3.3 Numerical method and discretizations

The parabolic problem eq.(3) with Dirichlet conditions has been solved using finite differences method for the derivative respect of time, and using finite element method respect of the space. Regarding spatial discretization, the selected domains corresponding to figure 3 are shown in figure 4. In order to solve the problem described by equation (3), first the

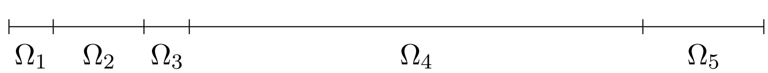


Figure 4: One-dimensional domains of spatial discretizacion

finite difference method is applied making use of a progressive scheme, as shown below:

$$\frac{\rho\, c_p\, (T(t+h) - T(t))}{h_t} - div(k \cdot \nabla T(t+h)) = f(x,t,s,T(t-s)) \tag{4}$$

Once a difference equation, which depends only of space, is gained equation (5), it is solved by means of lineal elements using Galerkin finite element method

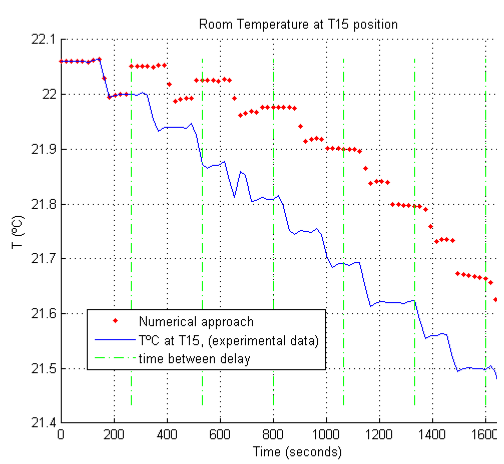
$$\int_\Omega \rho\, c_p\, W \cdot \varphi + h_t \int_\Omega k\, \nabla W \cdot \nabla \varphi\, dx = \int_\Omega F \cdot \varphi\, dx, \tag{5}$$

where $W = T - T_{|\partial\Omega}$. The function F collects all right hand side terms that appear after discretization, and φ is a test function that belongs to the basis of the finite elements.

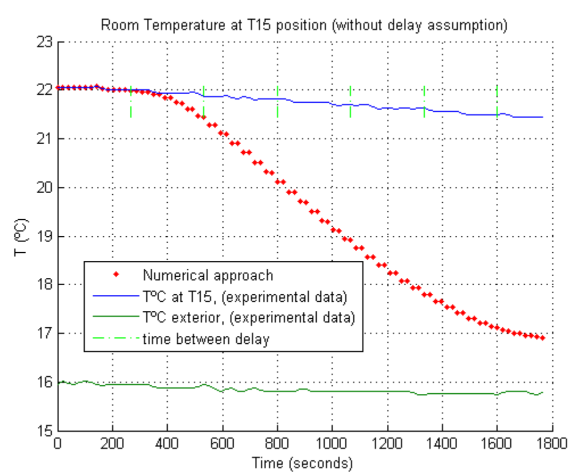
4 Results discussion

To evaluate the performance of the developed model, a comparison between the numerical data provided by the model and the experimental data is carried out. As an example, the numerical and experimental temperature values for T15 temperature sensor (which is located at 15 cm from the inner glass inside the box) are shown in figure 5.

When heat water is provided by including the time delay in the heat source term, the numerical temperature values are very close to experimental results, as can be seen in fig. 5a. However, when heat source term with time delay is not considered, which means that there is no heat source, the inside temperature values tend to external temperature as expected.



(a) Model with time delay in source term



(b) Model without source term

Figure 5: Room temperature response for T15 temperature sensor

It is interesting to highlight the shape likeness of temperature curves shown in figure 5a, which adopt the same pattern. This points out the suitability of the time delay considered effect.

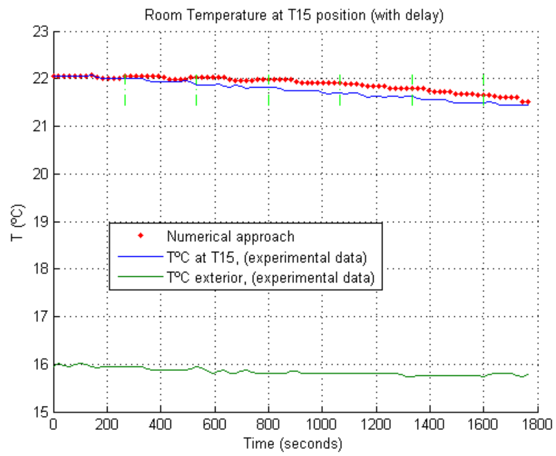


Figure 6: Comparison between numerical and gained temperature from T15 sensor

In relation with the small difference between the curves of figure 5a (note the y-axis scale), it could be connected to heat losses, which have not been considered.

5 Conclusions

The simplified *time delay* developed model provides a good level of coincidence between the numerical and the experimental data, despite of being a one-dimensional model. The effect of *time delay* included in the *source therm* offers good performance for temperature values inside the box, as shown by the shapes of the numerical and experimental data curves observed in figure 5a.

In order to get even better performance, both heat losses and other heat transfers mechanisms could be considered. Therefore further investigation is required.

Acknowledgements

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